

第廿七届全国华文独中数理学识比赛 数学科

1. 椭圆  $4x^2+y^2=16$  上点  $(1,2\sqrt{3})$  处之切线斜率为

The slope of the tangent to the ellipse  $4x^2+y^2=16$  at the point  $(1,2\sqrt{3})$  is

- (A)  $-\frac{2}{\sqrt{3}}$  (B)  $\frac{2}{\sqrt{3}}$  (C)  $-\frac{\sqrt{3}}{2}$  (D)  $\frac{\sqrt{3}}{2}$  (E) 以上皆非(none of the above)

2.  $\int_1^{e^e} \frac{\sqrt{1+\ln x}}{x} dx =$  \_\_\_\_\_

- (A)  $\frac{2}{3}$  (B)  $\frac{4}{3}$  (C)  $\frac{2}{3}(2\sqrt{2}-1)$  (D)  $\frac{2}{3}(\sqrt{2}-1)$  (E) 以上皆非(none of the above)

3.  $\int_0^{\pi} \cos^2 x dx =$  \_\_\_\_\_

- (A)  $\frac{\pi}{4}$  (B)  $\frac{\pi}{2}$  (C)  $1 - \frac{\pi}{4}$  (D)  $2 - \frac{\pi}{2}$  (E) 以上皆非(none of the above)

4.  $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx =$  \_\_\_\_\_

- (A)  $2\cos\sqrt{x} + C$  (B)  $-2\cos\sqrt{x} + C$  (C)  $\cos\sqrt{x} + C$  (D)  $\frac{1}{2}\cos\sqrt{x} + C$

(E) 以上皆非(none of the above)

5. 曲线  $y = x^2$  与  $y^2 = x$  所围区域面积为

The area enclosed between the curves  $y = x^2$  and  $y^2 = x$  is

- (A)  $\frac{1}{2}$  (B)  $\frac{2}{3}$  (C)  $\frac{1}{3}$  (D)  $\frac{1}{4}$  (E) 以上皆非(none of the above)

6. 火车以每秒 30 公尺速度经号志灯时，司机瞬时踩煞车，40 秒后完全停止，若其速度函数为  $v(t) = 30 - 0.75t$ ，则由煞车起算至车完全停止，车经过几公尺？

A train travelling at 30 meters per second passes a signal and the driver immediately applies the brakes. 40 seconds later the train grinds to halt. The velocity is given by

$v(t) = 30 - 0.75t$ . The distance the train travels in coming to rest in meters is

- (A) 1200 (B) 800 (C) 600 (D) 200 (E) 以上皆非(none of the above)

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7. 以每分钟 8 立方公分的水倒入半径 6 公分之正圆柱体，则柱内水每分钟升高几公分？  
Water is flowing into a vertical cylindrical tank of radius 6 cm at the rate of  $8 \text{ cm}^3/\text{minute}$ .  
How fast is the level of water in the tank rising in  $\text{cm}/\text{minute}$ ?
- (A)  $\frac{1}{9\pi}$       (B)  $\frac{2}{9\pi}$       (C)  $\frac{3}{9\pi}$       (D)  $\frac{4}{9\pi}$       (E) 以上皆非(none of the above)
8. 一制造商每周生产  $x$  公吨产品之成本为  $0.05x^2 + 2x + 300$  元，若每吨以价格  $p$  元卖出，  
则每周可卖出  $100 - 2p$  吨。问价格定为多少时可得最大利润？  
A manufacturer can produce a weekly output of  $x$  tones of his product at a total weekly cost  
of  $0.05x^2 + 2x + 300$  dollars. If he sells at a price of  $p$  dollars per tone, the weekly sale  
will be  $100 - 2p$  tones. How many dollars is the price at which his profit will be a  
maximum?
- (A)  $\frac{310}{11}$       (B) 28      (C)  $\frac{320}{11}$       (D) 30      (E) 以上皆非(none of the above)
9. 若  $\tan(x + y) = \frac{3}{4}$ ,  $\tan x = \frac{1}{2}$ , 则  $\tan y =$  \_\_\_\_\_  
If  $\tan(x + y) = \frac{3}{4}$ , and  $\tan x = \frac{1}{2}$ , then  $\tan y =$  \_\_\_\_\_
- (A)  $\frac{1}{2}$       (B) 1      (C)  $\frac{1}{11}$       (D)  $\frac{2}{11}$       (E) 以上皆非(none of the above)
10. 若  $\sin\left(x - \frac{\pi}{4}\right) = 3 \cos\left(x + \frac{\pi}{4}\right)$ , 则  $\tan x =$  \_\_\_\_\_  
If  $\sin\left(x - \frac{\pi}{4}\right) = 3 \cos\left(x + \frac{\pi}{4}\right)$ , then  $\tan x =$  \_\_\_\_\_
- (A)  $\frac{1}{2}$       (B) 1      (C)  $\frac{1}{3}$       (D) 0      (E) 以上皆非(none of the above)
11.  $(x^2 - \frac{1}{x})^{10}$  展式中  $x^{-4}$  之系数为  
The coefficient of  $x^{-4}$  in the expansion of  $(x^2 - \frac{1}{x})^{10}$  is
- (A) 32      (B) 42      (C) 52      (D) 45      (E) 以上皆非(none of the above)

12.  $\sum_{n=1}^{\infty} \frac{1}{n(2n+2)} =$

- (A)  $\frac{1}{2}$     (B)  $\frac{1}{4}$     (C)  $\frac{1}{6}$     (D)  $\frac{1}{8}$     (E) 以上皆非(none of the above)

13.  $\lim_{x \rightarrow 0} \frac{\sin x^2}{x} =$  \_\_\_\_\_

- (A)  $\infty$     (B) 1    (C) 0    (D)  $-\infty$     (E) 以上皆非(none of the above)

14. 设  $f(x) = \frac{\sin x}{1 + \cos x}$ , 则  $f'(\frac{\pi}{2}) =$  \_\_\_\_\_

Let  $f(x) = \frac{\sin x}{1 + \cos x}$ . Then  $f'(\frac{\pi}{2}) =$  \_\_\_\_\_

- (A)  $\frac{1}{2}$     (B) 1    (C)  $\frac{1}{4}$     (D)  $\frac{1}{3}$     (E) 以上皆非(none of the above)

15. 设  $\log_x 125 = 2$ , 则  $x =$  \_\_\_\_\_

Let  $\log_x 125 = 2$ . Then  $x =$  \_\_\_\_\_

- (A) 5    (B) 5.5    (C)  $5\sqrt{5}$     (D)  $3\sqrt{5}$     (E) 以上皆非(none of the above)

16. 设  $\log_a \frac{1-a}{2} = 2$ , 则  $a =$  \_\_\_\_\_

Let  $\log_a \frac{1-a}{2} = 2$ . Then  $a =$  \_\_\_\_\_

- (A) 2    (B)  $\frac{1}{2}$     (C)  $\frac{1}{3}$     (D)  $\frac{1}{4}$     (E) 以上皆非(none of the above)

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17. 设  $x_1, x_2, \dots, x_n$  为任意  $n$  个实数,  $\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$ , 则  $\sum_{i=1}^n (x_i - \bar{x})^2 =$  \_\_\_\_\_

Let  $x_1, x_2, \dots, x_n$  be  $n$  arbitrary real numbers and  $\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$ . Then  $\sum_{i=1}^n (x_i - \bar{x})^2 =$  \_\_\_\_\_

- (A)  $\sum_{i=1}^n x_i^2 - n\bar{x}^2$       (B)  $\sum_{i=1}^n x_i^2 - \bar{x}^2$       (C)  $\sum_{i=1}^n x_i^2 - n^2\bar{x}^2$   
 (D)  $\sum_{i=1}^n x_i^2 - \frac{\bar{x}^2}{n}$       (E) 以上皆非(none of the above)

18. 函数  $f(x, y) = 2x + 3y + 7$  在  $0 \leq x \leq 4, 0 \leq y \leq 3$  范围内之最大值为

The maximum value of  $f(x, y) = 2x + 3y + 7$  on the region  $0 \leq x \leq 4, 0 \leq y \leq 3$  is

- (A) 7      (B) 14      (C) 12      (D) 24      (E) 以上皆非(none of the above)

19. 已知方程式  $x^3 - 2x^2 + 5x - 3 = 0$  之三根为  $\alpha, \beta, \gamma$ , 则  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$  为

Let  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 - 2x^2 + 5x - 3 = 0$ . Then  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$  is

- (A)  $\frac{5}{3}$       (B)  $-\frac{5}{3}$       (C) -13      (D) 13      (E) 以上皆非(none of the above)

20. 设  $w = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ , 则  $w^9 + w^{18} =$  \_\_\_\_\_

Let  $w = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ . Then  $w^9 + w^{18} =$  \_\_\_\_\_

- (A) -1      (B) 2      (C) 1      (D) -2      (E) 以上皆非(none of the above)

21. 某学生做一份有 15 题多重选择题之考卷。设每题答错机率为 0.1, 则该学生答对超过 90% 考题之机率为

A student sits a multi-choice exam with 15 questions, each of which he has 0.1 chance of getting wrong. The probability he will exceed 90% in this test is

- (A)  $(0.1)(0.9)^{14}$       (B)  $\binom{15}{1}(0.1)(0.9)^{14}$       (C)  $(0.9)^{15}$   
 (D)  $(0.9)^{15} + \binom{15}{1}(0.1)(0.9)^{14}$       (E) 以上皆非(none of the above)

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22. 一罐中有 4 黑球 1 白球，随机抽取一球不放回。再随机抽取一球，则第二次抽到黑球之几率为

A jar contains 4 black balls and one white ball. One ball is randomly withdrawn with its color being noted. Without replacing this ball, another is withdrawn and the color is noted. The probability of drawing a black ball on the second draw is

- (A)  $\frac{2}{3}$     (B)  $\frac{3}{4}$     (C)  $\frac{17}{20}$     (D)  $\frac{4}{5}$     (E) 以上皆非(none of the above)

23. 一班 15 个学生修英文，数学及自然科，每科期末颁一奖给最好学生。若每位学生至多只能获颁两个奖项，则学生得奖的情况共有多少种？

There are 15 students in a class. They all take English, Mathematics and Science. At the end of the semester, one prize is awarded to the best student of each subject. How many ways could the prizes be awarded if no student can win more than two prizes?

- (A)  $14^3$     (B)  $14 \times 15^2$     (C)  $15^3 - 15$     (D)  $15 \times 14 \times 13$   
(E) 以上皆非(none of the above)

24. 某人得一种传染病，此人传染给  $N$  个人知机率分别为  $P\{N = 0\} = 0.1$ ,  $P\{N = 1\} = 0.25$ ,  $P\{N = 2\} = 0.4$ ,  $P\{N = 3\} = 0.2$ ,  $P\{N = 4\} = 0.05$ 。则  $N$  的变异数为何？

A person infected with a certain disease can pass it on to others. Let  $N$  be the number of others infected by this person. The probabilities of  $N$  are respectively  $P\{N = 0\} = 0.1$ ,  $P\{N = 1\} = 0.25$ ,  $P\{N = 2\} = 0.4$ ,  $P\{N = 3\} = 0.2$ ,  $P\{N = 4\} = 0.05$ . What is the variance of  $N$ ?

- (A) 1.0275    (B) 1.8514    (C) 2.3020    (D) 1.5165  
(E) 以上皆非(none of the above)

25. 某种细菌寄养于布丁中依  $\frac{dy}{dt} = Ky$  之模式成长，其中  $y$  表时间  $t$  天时之细菌数目， $K$  为常数。若  $t=0$  时， $y = 50$ ，且细菌以每天 1000 个数目成长。则  $K$  值为何？

The rate of increase per day in the number of bacteria in a pudding is according to the model

$\frac{dy}{dt} = Ky$ , where  $y$  is the number of bacteria at day  $t$ ,  $K$  is a constant. If  $y = 50$  when

$t=0$ , and they are increasing at the rate of 1000 per day, what is the value of  $K$  ?

- (A) 50    (B) 30    (C) 20    (D) 10    (E) 以上皆非(none of the above)