第 25 届全国华文独中数理学识比赛（2008）
数学学科

1. 解不等式 $2^{x^2-2x-3} < (\frac{1}{2})^{3(x-1)}$

   Solve the inequality $2^{x^2-2x-3} < (\frac{1}{2})^{3(x-1)}$

   (A) -3≤ x < 2　　(B) x <-3， x > 2
   (C) -3< x ≤ 2　　(D) -3< x < 2
   (E) 以上皆非 None of the above

2. 解方程 $\sin 5x = \sin 4x$
   Solve the equation $\sin 5x = \sin 4x$

   (A) $\left\{ x \mid x = \frac{4}{9}k\pi \pm \frac{\pi}{9}, k \in \mathbf{Z} \right\}$

   (B) $\{ x \mid x = 2k\pi, k \in \mathbf{Z} \}$

   (C) $\{ x \mid x = k\pi, k \in \mathbf{Z} \}$

   (D) $\left\{ x \mid x = \frac{4}{9}k\pi \pm \frac{\pi}{9}, \text{或} x = 2k\pi, k \in \mathbf{Z} \right\}$

   (E) 以上皆非 None of the above

   此处 $\mathbf{Z} = \{0, \pm 1, \pm 2, \ldots\ldots\ldots\}$

3. 已知一个直角三角形的三条边长成等差数列，则三边的边长的比例为?

   Given the lengths of the three sides of a right angle triangle form an arithmetical sequence, what is the ratio of the three sides?

   (A) 1:2:3　　(B) 2:3:4　　(C) 3:4:5　　(D) 8:10:12
   (E) 以上皆非 None of the above

4. 某药厂今年制药 5 吨，如果平均每年的产量比上一年增加 10%，那么从今年起，几年内可使总产量达到 30 吨？

A medical factory produces 5 tons of medicine in this year. If the production increases 10% each year, then starting from this year, how many years will it take for the production to reach an amount of 30 tons?

(A) 4 (B) 5
(C) 6 (D) 7
(E) 8
(log 1.6=0.2041，log 1.1=0.0414)

5. 设 $a>0$, $a\neq1$ 且 $A=|\log_a(1-x)|$, $B=|\log_a(1+x)|$, 当 $x\in(0,1)$, 列何者为真？

Let $a>0$, $a\neq1$ and $A=|\log_a(1-x)|$, $B=|\log_a(1+x)|$ of $x\in(0,1)$, which of the following statement is true?

(A) A=B (B) A<B
(C) A>B (D) A⩾B
(E) 以上皆非 None of the above

6. 若 $a,b>0$ 且 $a+b=1$， 则 $(a+\frac{1}{a})(b+\frac{1}{b})$ 的最小值为?

If $a,b>0$ and $a+b=1$, what is the minimum value of $(a+\frac{1}{a})(b+\frac{1}{b})$?

(A) 6 (B) $\frac{25}{4}$
(C) $\frac{27}{4}$ (D) 4
(E) $\frac{4}{27}$

7. 函数 $f(x)=\sqrt{\frac{1}{2}-x}+\sqrt{x-\frac{1}{3}}$ 的最大值为 a，最小值为 b，则 a+b= ？

If the maximum and minimum values of the function $f(x)=\sqrt{\frac{1}{2}-x}+\sqrt{x-\frac{1}{3}}$ are a and b respectively, then a+b= ?

(A) $\frac{\sqrt{3}}{6}(2+\sqrt{2})$ (B) $\frac{\sqrt{2}}{6}(3+\sqrt{3})$ (C) $\frac{1}{\sqrt{6}}$ (D) $\frac{1}{\sqrt{3}}$ (E) 0

8. 已知 $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdots\cdots\frac{99}{100}<x$，从以下的数值之中找出 x 之最小值。

Given $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}\cdots\cdots\frac{99}{100}<x$, among the following values, find the smallest value of x.

(A) $\frac{1}{10000}$ (B) $\frac{1}{1000}$ (C) $\frac{1}{100}$ (D) $\frac{1}{10}$

(E) 以上皆非 None of the above

9. 过点($\sqrt{2}$, 0)的所有直线中且通过两个不同的有理点的直线有__条。

How many straight lines which will pass through the given point ($\sqrt{2}$, 0) and two other rational points?

(A) 0
(B) 1
(C) 至少两条 at least 2 straight lines
(D) 无限条 infinitely many straight lines
(E) 以上皆非 None of the above

(rational point: $(\frac{a}{b},\frac{m}{n})$, a, b, m, n $\in \mathbf{Z}$, b, n$\neq 0$)

10. 已知二次方程式 $x^2+xy-6y^2-20x-20y+k=0$ 表示两条直线，这两条直线的夹角是？

Given a quadratic equation $x^2+xy-6y^2-20x-20y+k=0$ represents a system of two straight lines, what is the angle between these two straight lines:

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

(E) 以上皆非 None of the above

11. 已知圆 $C: x^2+y^2-2x+4y-4=0$，求直线 $l: y=x+b$ 使 $l \cap C$ 有二点 A,B 且以 AB 为直径的圆过原点(0,0)，则 l:

Given a circle $C: x^2+y^2-2x+4y-4=0$, find the straight line $l: y=x+b$ such that the intersection of $l \cap C$ has two point A,B and the circle with AB as diameter also passes through the origin(0,0), then l:

(A) y = x-4 , y = x-1 (B) y = x+4 , y = x-1

(C) y = x+4 , y = x+1 (D) y = x-4 , y = x+1

(E) 以上皆非 None of the above

12. 已知直线 x+2y=3 交圆 $x^2+y^2+x-6y+a=0$ 於 P,Q. O 为原点,若 OP⊥OQ,则 a = ？

Given straight line x+2y=3 intersects the circle $x^2+y^2+x-6y+a=0$ at two points P and Q. Let O be the origin. If OP⊥OQ, then a = ?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

13. 把圆 $x^2+(y-1)^2=1$ 与椭圆 $9x^2+(y+1)^2=9$ 公交点用直线连结起来得到的图形为?

If we use straight lines to join the intersection points of the circle $x^2+(y-1)^2=1$ and the ellipse $9x^2+(y+1)^2=9$, what is the shape of the diagram?

(A) 四边形 Quadrilateral (B) 长方形 Rectangle
(C) 三角形 Triangle (D) 正三角形 Equilateral triangle
(E) 以上皆非 None of the above

14. 已知椭圆 $\frac{x^2}{9}+\frac{y^2}{4}=1$， 直线 $x+2y+18=0$。试在椭圆上求一点 P，使从 P 至直线的距离为最短 l，求 $l=$____。

Given an ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, and a straight line $x+2y+18=0$. Let P be a point on the ellipse, and l be the distance between P and the straight line. Find the smallest value of l.

(A) 4 (B) $\frac{\sqrt{13}}{5}$ (C) $\sqrt{\frac{13}{5}}$ (D) 6 (E) $\frac{13\sqrt{5}}{5}$

15. 求与圆 $x^2+y^2-4x-8y+15=0$ 相切於点 P(3,6)且过点 Q(5,6)的圆方程式。

Find the equation of a circle while passes through a given point Q(5,6) and tangent to the given circle $x^2+y^2-4x-8y+15=0$ at the point P(3,6).

(A) $x^2+y^2-8x-16y+25=0$ (B) $x^2+y^2-8x-16y+50=0$
(C) $x^2+y^2-8x-16y+75=0$ (D) $x^2+y^2-8x-16y+100=0$
(E) 以上皆非 None of the above

16. 已知 $n \in \mathrm{N} = \{1, 2, 3, \ldots, n, \ldots\}$，计算 $\frac{1+i^{3n}+i^{5n}+i^{7n}+\ldots+i^{1997n}}{1 \cdot i^{3n} \cdot i^{5n} \cdots i^{1997n}} =$ ____.
此处 $i = \sqrt{-1}$.

Given $n \in \mathrm{N} = \{1, 2, 3, \ldots, n, \ldots\}$, compute $\frac{1+i^{3n}+i^{5n}+i^{7n}+\ldots+i^{1997n}}{1 \cdot i^{3n} \cdot i^{5n} \cdots i^{1997n}} =$ ____.
Here $i = \sqrt{-1}$.

(A) 998 (B) -998 (C) 997 (D) -997 (E) 1,999,-997

17. 设 x,y,z 为正实数且 x+y+z=1，则 $\frac{1}{x} + \frac{4}{y} + \frac{9}{z}$ 的最小值是？

Let x,y,z be the positive real numbers and x+y+z=1, then the minimum value of $\frac{1}{x} + \frac{4}{y} + \frac{9}{z}$ is?

(A) 42 (B) 54 (C) 36 (D) 27
(E) 以上皆非 None of the above

18. 求 $\lim_{x \to 1} \frac{1-x^3}{2-\sqrt{x^2+3}} =$ ____.
Find $\lim_{x \to 1} \frac{1-x^3}{2-\sqrt{x^2+3}} =$ ____.

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

19. 求 $\lim_{n \to \infty} \frac{10^n}{n!} (n! = 1 \times 2 \times 3 \times \ldots \times n)$

Find $\lim_{n \to \infty} \frac{10^n}{n!} (n! = 1 \times 2 \times 3 \times \ldots \times n)$

(A) 0 (B) 1 (C) 2 (D) ∞
(E) 以上皆非 None of the above

20. 设 $f(x)=\frac{3+x}{3-x}$，求 $f'(2)=$____ 。

Let $f(x)=\frac{3+x}{3-x}$， find $f'(2)=$____ .

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

21. 己知 f(x+y)=f(x)+f(y)+2xy 对所有 x,y∈R 且 f′(0)=0. 求 f(x)=__

Given f(x+y)=f(x)+f(y)+2xy for all x,y∈R and f′(0)=0. Find f(x)=__.

(A) x^2 (B) x^2+1 (C) x^2-1 (D) x^2+x+1
(E) 以上皆非 None of the above

22. 一位农夫用 1000 尺的围栏在一直线的河流边上去围一个长方形，农夫只围三边而不用围靠河的边上，问所围的最大面积为____平方尺。

A farmer with 1000 feet of fencing wishes to enclose a rectangular field adjacent to a straight river. If the farmer does not fence the side along the river, find the largest area that he can fence.

(A) 998 (B) 4950 (C) 9800 (D) 80000 (E) 125000

23. 求 $\int_1^2 \frac{x^3-x^2+1}{x^2}dx=$______ .

Find $\int_1^2 \frac{x^3-x^2+1}{x^2}dx=$______ .

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

24. 求由两曲线 C_1:y=f(x)=4x 及 C_2:y=g(x)=x^3 所围的面积。

Find the area of the region bounded by the two curves C_1:y=f(x)=4x and C_2:y=g(x)=x^3.

(A) 0 (B) 4 (C) -4 (D) 8 (E) -8

25. 求 $\int \frac{\sin x}{(1+\cos x)^3}dx =$ ____ .

Find $\int \frac{\sin x}{(1+\cos x)^3}dx =$ ____ .

(A) $\frac{-1}{2(1+\cos x)^2}+C$ (B) $\frac{1}{2(1+\cos x)^2}+C$ (C) $\frac{1}{2(1+\sin x)^2}+C$

(D) $\frac{-1}{2(1+\sin x)^2}+C$ (E) 以上皆非 None of the above

26. 求由 y=x^2+2, X-轴及直线 x=-2 与 x=3 所围的面积绕 X-轴旋转一周的旋转体积为___立方单位。

The region bounded by the graph of y=x^2+2, the X-axis, and the lines x=-2 and x=3 is revolved about the X-axis. Find the volume of the resulting solid=____ cubic units.

(A) $\frac{361\pi}{3}$ (B) $\frac{362\pi}{3}$ (C) $\frac{121\pi}{3}$ (D) $\frac{364\pi}{3}$ (E) $\frac{365\pi}{3}$

27. 六名学生排成一排，其中某甲不能排头也不能排尾，一共有多少种不同的排法？

Six students join in a straight line, one special student out of the six cannot be placed in the first and the last position. How many different ways of arrangements of standing in a straight line?

(A) 240 (B) 320 (C) 400 (D) 480 (E) 560

28. 求$\sum_{i=1}^{100} i(i+1) =$ ____ 。

Find$\sum_{i=1}^{100} i(i+1) =$ ____ .

(A) 343000 (B) 343100 (C) 343200 (D) 343300 (E) 343400

29. 若方程组$\begin{cases} 2x-y-z=0 \\ 3x+y-4z=0 \\ 5x-2y+mz=0 \end{cases}$ 有异於$(0,0,0)$的解，则 m= __。

If the system of equations $\begin{cases} 2x-y-z=0 \\ 3x+y-4z=0 \\ 5x-2y+mz=0 \end{cases}$ has nontrivial solution (that means solution is different from (0,0,0), m=_____.

(A) 1 (B) 2 (C) 3 (D) -3 (E) -2

30. 球面 S:$x^2+y^2+z^2+2x+6z+k=0$, 直线 L: $\frac{x+2}{2}=\frac{y-2}{1}=\frac{z-3}{1}$ ，若球面 S 与 L 相切，则 k=__

The surface equation of a sphere is S:$x^2+y^2+z^2+2x+6z+k=0$, while that of a straight line is L: $\frac{x+2}{2}=\frac{y-2}{1}=\frac{z-3}{1}$. If S touches L, then k=__.

(A) 15 (B) 20 (C) 25 (D) -20 (E) -25

答案:

1. (D)
2. (D)
3. (C)
4. (B)
5. (C)
6. (B)
7. (A)
8. (D)
9. (B)
10. (B)
11. (D)
12. (C)
13. (D)
14. (E)
15. (C)
16. (E)
17. (C)
18. (D)
19. (A)
20. (E)
21. (A)
22. (E)
23. (B)
24. (D)
25. (B)
26. (E)
27. (D)
28. (E)
29. (D)
30. (E)