

1. 考慮數據 5,6,3,8,5,9。令  $\sigma$  為其標準差，而  $m$  為其平均差。求  $|\sigma^2 - m|$  為何？  
Consider the data 5,6,3,8,5,9. Let  $\sigma$  be the standard deviation and  $m$  the mean deviation (average deviation) of the data. What is the absolute value  $|\sigma^2 - m|$  ?

- (a) 0
- (b)  $\frac{1}{2}$
- (c)  $\frac{5}{2}$
- (d)  $\frac{7}{3}$
- (e) 以上皆非 None of the above

2. 求函數  $f(x) = (3x - 1)^3(x^2 + 5)^2$  在  $x = 0$  的微分值。  
Find the derivative of the function  $f(x) = (3x - 1)^3(x^2 + 5)^2$  at  $x = 0$ .

- (a) 0
- (b) 25
- (c) 30
- (d) 225
- (e) 以上皆非 None of the above

3. 令函數  $y = x^2 - 3x - 4$  在  $(3, -4)$  的切線為  $L$ 。求  $L$  與  $x$ -軸的交點的  $x$  座標值。  
Let  $L$  be the tangent line of the curve  $y = x^2 - 3x - 4$  at  $(3, -4)$ . Find the  $x$  coordinate of the point of intersection of  $L$  and the  $x$ -axis.

- (a)  $\frac{-13}{3}$
- (b) -13
- (c) 0
- (d) 3
- (e) 以上皆非 None of the above

4. 下列哪一個方程式代表一條與橢圓  $x^2 + 4y^2 = 5$  相切的直線?

Which of the following equations represents a tangent line to the ellipse  $x^2 + 4y^2 = 5$  ?

(a)  $x + y = \frac{-5}{2}$

(b)  $x + y = \frac{-3}{2}$

(c)  $x + y = \frac{1}{2}$

(d)  $x + y = \frac{3}{2}$

(e) 以上皆非 None of the above

5. 請問有幾種不同的方法可以將 35 寫成兩個以上的連續正整數的和。

How many different ways are there to write 35 as a sum of consecutive positive integers.

(a) 2

(b) 4

(c) 6

(d) 8

(e) 以上皆非 None of the above

6. 以下何者為  $1 + i$  的四次方根?

Which of the following is a 4<sup>th</sup> root of  $1 + i$ ?

(a)  $\sqrt[4]{2} \left( \cos \frac{3\pi}{16} + i \sin \frac{3\pi}{16} \right)$

(b)  $\sqrt[4]{2} \left( \cos \frac{5\pi}{16} + i \sin \frac{5\pi}{16} \right)$

(c)  $\sqrt[4]{2} \left( \cos \frac{7\pi}{16} + i \sin \frac{7\pi}{16} \right)$

(d)  $\sqrt[4]{2} \left( \cos \frac{9\pi}{16} + i \sin \frac{9\pi}{16} \right)$

(e) 以上皆非 None of the above

7.敘述  $|x + 1| + 3|x - 2| < 6$  與下列何者等值?

Which of the following statement is equivalent to  $|x + 1| + 3|x - 2| < 6$ ?

- (a)  $\frac{1}{2} < x < 3$
- (b)  $\frac{1}{2} < x < \frac{11}{4}$
- (c)  $1 < x < 3$
- (d)  $1 < x < \frac{11}{4}$
- (e) 以上皆非 None of the above

8.令  $A$  爲一菱形而  $B$  爲一正方形。如果  $A$  與  $B$  具有等長的邊，但是  $A$  的面積只有  $B$  的一半，則  $A$  的長對角線與短對角線的長度比爲何?

Let  $A$  be a rhombus and  $B$  a square. If  $A$  and  $B$  have the same length for their sides, but the area of  $A$  is half of that of  $B$ , what is the ratio of the length of the long diagonal of  $A$  to that of the short diagonal of  $A$ ?

- (a)  $2 - \sqrt{3}$
- (b)  $\frac{1}{2}$
- (c)  $\sqrt{3}$
- (d)  $2 + \sqrt{3}$
- (e) 以上皆非 None of the above

9.令  $C$  爲由方程式  $4x^2 + 4y^2 - 20x + 4y + 18 = 0$  所定義出來的圓，而  $L$  爲過原點  $O(0,0)$  且與  $C$  相切於點  $B$  的直線。求點  $B$  與  $O$  的距離。

Let  $C$  be the circle given by the equation  $4x^2 + 4y^2 - 20x + 4y + 18 = 0$ , and let  $L$  be a straight line passing through the origin  $O(0,0)$  and tangent at a point  $B$  on the circle  $C$ . Find the distance between  $O$  and  $B$ .

- (a)  $\frac{\sqrt{6}}{2}$
- (b)  $2$
- (c)  $\frac{\sqrt{18}}{2}$
- (d)  $\frac{\sqrt{21}}{2}$
- (e) 以上皆非 None of the above

10. 令  $x, y, z$  為三個未知元。在展開  $(x + y + z)^5$  後  $x^2yz$  的係數為何?

Let  $x, y, z$  be indeterminates. After expanding, what is the coefficient of  $x^2yz$  in  $(x + y + z)^5$ ?

- (a) 250
- (b) 500
- (c) 1000
- (d) 2000
- (e) 以上皆非 None of the above

11. 一個盒子中有兩個新球以及三個舊球。如果連續由盒中取出兩個球(一次一個，不放回)，則拿到兩個新球的機率是多少?

There are 5 balls in a box with two of them new and three of them old. If we take two balls out from the box, one at a time without putting them back into the box, then what would be the probability of getting two new balls?

- (a)  $\frac{1}{20}$
- (b)  $\frac{1}{10}$
- (c)  $\frac{3}{20}$
- (d)  $\frac{3}{10}$
- (e) 以上皆非 None of the above

12. 假設  $x, y, z$  為正實數，且滿足  $xy = z, xz = 2y$ , 及  $yz = 4x$ 。下列何者是  $x + y + z$ ?

Suppose that  $x, y, z$  are positive real numbers satisfying  $xy = z, xz = 2y$ , and  $yz = 4x$ . Which of the following is  $x + y + z$ ?

- (a)  $2 + \sqrt{2}$
- (b)  $2 + 2\sqrt{2}$
- (c)  $2 + 3\sqrt{2}$
- (d)  $3 + 2\sqrt{2}$
- (e) 以上皆非 None of the above

13. 令  $(x, y, z)$  為聯立方程組  $\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 20 \\ 3x - 2y - 5z = 6 \end{cases}$  的解。求  $x + y + z$

Let  $(x, y, z)$  be the solution of the system of equations  $\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 20 \\ 3x - 2y - 5z = 6 \end{cases}$ . Then

$x + y + z$  is

- (a) 14
- (b) 15
- (c) 16
- (d) 17
- (e) 以上皆非 None of the above

14. 令  $A = \begin{pmatrix} 1 & -1 & 1 \\ a & 1 & 3 \\ 3 & 4 & b \end{pmatrix}$ 。下列何者會使  $A$  成為可逆矩陣?

Let  $A = \begin{pmatrix} 1 & -1 & 1 \\ a & 1 & 3 \\ 3 & 4 & b \end{pmatrix}$ . Which of the following makes  $A$  an invertible matrix?

- (a)  $(a, b) = (0, 24)$
- (b)  $(a, b) = (6, 0)$
- (c)  $(a, b) = (1, 10)$
- (d)  $(a, b) = (10, 1)$
- (e) 以上皆非 None of the above

15. 令  $y = f(x)$  為定義於閉區間  $[a, b]$ ,  $a < b$ , 且對所有  $x \in [a, b]$ ,  $f(x) > 0$ 。由曲線  $y = f(x)$ ,  $y = 0$ ,  $x = a$  及  $x = b$  所圍住的區域記為  $A$ 。下面哪一個式子表示區域  $A$  繞  $x$ -軸旋轉所得的旋轉體的體積?

Let  $y = f(x)$  be a continuous function defined on the closed interval  $[a, b]$ ,  $a < b$ , with  $f(x) > 0$  for all  $x \in [a, b]$ . Denote by  $A$  the region bounded by the curve  $y = f(x)$ ,  $y = 0$ ,  $x = a$ , and  $x = b$ . Which of the following gives the volume of the solid generated by revolving  $A$  along the  $x$ -axis?

- (a)  $\frac{4\pi}{3} f(x)^3$   
 (b)  $\int_a^b \frac{4\pi}{3} f(x)^3 dx$   
 (c)  $\int_a^b \pi f(x)^2 dx$   
 (d)  $\int_a^b 2\pi f(x) dx$   
 (e) 以上皆非 None of the above

16. 函數  $y = x^2 e^x$  與  $x$ -軸在  $0 \leq x \leq 1$  範圍內所圍住的區域面積為

The area of the region between the function  $y = x^2 e^x$  and the  $x$ -axis for  $0 \leq x \leq 1$  is

- (a) 0  
 (b)  $e - 2$   
 (c)  $e - 1$   
 (d)  $2e - 1$   
 (e) 以上皆非 None of the above

17. 令  $f(x) = \frac{2x-1}{x-1}$ 。下面何者為  $f(x)$  的圖形的漸進線?

Let  $f(x) = \frac{2x-1}{x-1}$ . Which of the following is an asymptote of the graph of  $f(x)$ ?

- (a)  $y = -2$   
 (b)  $x = 1$   
 (c)  $y = 1$   
 (d)  $x = 2$   
 (e) 以上皆非 None of the above

18. 假設  $a$  是一個實數使得函數  $f(x) = a \sin x + \frac{1}{3} \sin 3x$  在  $x = \frac{\pi}{3}$  處有極值。則  $a$  為何? 而此極值是極大值還是極小值?

Suppose that  $a$  is a real number such that the function  $f(x) = a \sin x + \frac{1}{3} \sin 3x$  has an

extremum at  $x = \frac{\pi}{3}$ . What is  $a$ ? Is the extremum a maximum or a minimum?

- (a)  $a = -2$ , 極小值 minimum      (b)  $a = -2$ , 極大值 maximum  
 (c)  $a = 2$ , 極小值 minimum      (d)  $a = 2$ , 極大值 maximum  
 (e) 以上皆非 None of the above

19. 令  $f(x) = x^3 - 3x + 3$ 。下列敘述何者正確？

Let  $f(x) = x^3 - 3x + 3$ . Which of the following statements is correct?

- (a) 其極大值為正, 極小值為負  
The local maximum of  $f(x)$  is positive and the local minimum is negative.
- (b) 其極大值為負, 極小值為正  
The local maximum of  $f(x)$  is negative and the local minimum is positive.
- (c) 極大值和極小值同為負  
Both of the local maximum and local minimum of  $f(x)$  are negative.
- (d) 極大值和極小值同為正  
Both of the local maximum and local minimum of  $f(x)$  are positive.
- (e) 以上皆非 None of the above

20. 下列何者最接近  $\sqrt{\frac{10}{11}}$  ?

Which of the following numbers is the closest to  $\sqrt{\frac{10}{11}}$  ?

- (a) 0.912
- (b) 0.930
- (c) 0.953
- (d) 0.971
- (e) 以上皆非 None of the above

21. 令  $a, b, c$  為一三角形的三邊邊長。假設  $ab + bc + ca = 2$  並令  $A = a + b + c$ 。下列何者為正確？

Let  $a, b$  and  $c$  be the lengths of the sides of a triangle. Suppose that  $ab + bc + ca = 2$ , and  $A = a + b + c$ . Which of the following is correct?

- (a)  $1 \leq A < \sqrt{5}$
- (b)  $\sqrt{2} \leq A < \sqrt{6}$
- (c)  $\sqrt{6} \leq A < \sqrt{10}$
- (d)  $\sqrt{10} < A < \sqrt{15}$
- (e) 以上皆非 None of the above

22. 三邊邊長為整數，且其最長邊的邊長是 10 的三角形有幾個？

How many different triangles are there with the lengths of the sides being integers, and the longest one being 10?

- (a) 25
- (b) 30
- (c) 40
- (d) 55
- (e) 以上皆非 None of the above

23. 現有四種顏色的球，每種顏色各兩個，將其排成一列。如果同樣顏色的球不能放在一起，可有幾種排列方式？

There are 8 balls in four different colors, with two balls in each color. If they are to be placed in a row, and no two of the same color are placed next to each other, how many ways can it be done?

- (a) 320
- (b) 482
- (c) 576
- (d) 864
- (e) 以上皆非 None of the above

24. 令  $m$  及  $n$  為正整數。如果  $\frac{m+n}{m^2+mn+n^2} = \frac{2}{13}$ ，則下列何者為  $m+n$ ？

Let  $m$  and  $n$  be positive integers. If  $\frac{m+n}{m^2+mn+n^2} = \frac{2}{13}$ , what could be  $m+n$ ?

- (a) 6
- (b) 7
- (c) 8
- (d) 9
- (e) 以上皆非 None of the above

25. 令  $x_1 = 3$ ，且當  $n \geq 1$  時  $x_{n+1} = x_n^2 - 2$ 。則  $\lim_{n \rightarrow \infty} \frac{x_{n+1}}{x_1 x_2 \dots x_n}$  為？

Let  $x_1 = 3$ , and  $x_{n+1} = x_n^2 - 2$  for  $n \geq 1$ . Then  $\lim_{n \rightarrow \infty} \frac{x_{n+1}}{x_1 x_2 \dots x_n}$  is?

- (a)  $\sqrt{5}$
- (b)  $\sqrt{6}$
- (c)  $\sqrt{7}$
- (d)  $\sqrt{8}$
- (e) 以上皆非 None of the above