馬來西亞留台成功大學校友會
主　辦
2024 年
第三十九屆成大數理比賽

考生指示：

（一）解答所有問題。

（二）將正確答案在答案紙上的圓圈內"塗黑"，每題只准給一個答案。

（三）正確的答案得三分，錯誤的答案扣一分，不做答的零分。

INSTRUCTIONS TO CANDIDATES
1.　**Attempt all questions.**

2.　**Pick the correct answer and make a mark "●" in the circle provided in the answer sheet. Only one answer is allowed for each question.**

3.　**Three marks for a correct answer, one mark will be deducted for each wrong answer. No mark will be given to each question not attempted.**

1. 在一種稱為旋轉器的嘉年華遊樂設施，如圖所示人可以站在一個繞著垂直軸旋轉的大圓柱體內的壁架上。 當它達到足夠高的轉速時，壁架就會掉落。若半徑為 2 米，旋轉週期為 2 秒，則人不滑落的最小摩擦係數為 (註：重力加速度 g=9.8 米/秒²)

   In a carnival ride called the rotator, people stand on a ledge inside a large cylinder that rotates about a vertical axis. When it reaches a high enough rotational speed, the ledge drops away. If the radius is 2 m and the period of rotation is 2 sec, then the minimum coefficient of friction for the people not to slide down is (Note: the acceleration of gravity g=9.8 m/s²)

   (A) 0.9　　(B) 0.8　　(C) 0.7　　(D) 0.6　　(E) 0.5
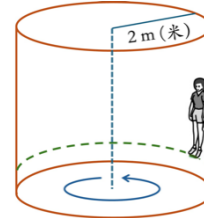
2. 塊A的質量為 m_A，最初靜止在水平光滑的地面上，質量為 m_B 的塊B則靜止在塊A上面，而兩塊之間的靜摩擦係數為 μ_s。若要塊A在塊B下方滑出，作用在塊A的水平拉力需大於：

   Block A, with mass m_A, is initially at rest on a horizontal smooth floor. Block B, with mass m_B, is initially at rest on the top surface of A. The coefficient of static friction between the two blocks is μ_s. Block A is pulled with a horizontal force. Block A will slide out from block B if the force is greater than:

   (A) $m_A g$　　(B) $m_B g$　　(C) $\mu_s m_A g$　　(D) $\mu_s m_B g$　　(E) $\mu_s (m_A + m_B) g$

3. 兩個相同的圓盤，每個圓盤的轉動慣量為 $I = \frac{1}{2} MR^2$，它們在水平地板上滾動而不滑動，然後各自滾上斜面。圓盤 A 在斜面只滾動無滑動，而圓盤 B是在無摩擦的斜面; 除此之外，兩斜面沒有其他差別。如果圓盤A離地面的最高高度是 12 公分，在相同的初始條件下，圓盤B離地面最高高度是多少？

   Two identical disks, each with a rotational inertia $I = \frac{1}{2} MR^2$, are rolling without slipping across a horizontal floor and then up an incline. Disk A rolls up the incline without sliding, while disk B rolls up a frictionless incline. The inclines are otherwise identical. If Disk A reaches a height of 12 cm above the floor before rolling down again, what height above the floor does Disk B reach?

   (A) 24 cm (公分)　　(B) 18 cm (公分)　　(C) 12 cm (公分)　　(D) 8 cm (公分)　　(E) 6 cm (公分)

4. 彈簧常數為 100 牛頓/米 的彈簧一端連接 2 公斤質量的物體，另一端連接 4 公斤質量物體。 將兩物體靜置非光滑水平表面上並將彈簧壓縮 10 公分。當壓縮彈簧被釋放時，物體會振盪運動。 當彈簧第一次達到平衡長度時，質量 2 公斤的物體速度為 0.4 米/秒。在此瞬間，損失的機械能為：

   A 2.0-kg mass is attached to one end of a spring with a spring constant of 100 N/m and a 4.0-kg mass is attached to the other end. The masses are placed on a horizontal surface and the spring is compressed 10 cm. The spring is released with the masses from rest and then the masses oscillate. The spring has its equilibrium length for the first time when the 2.0-kg mass has a speed of 0.4 m/s. The mechanical energy lost at this moment is:

   (A) 0 J (焦耳)　　(B) 0.26 J (焦耳)　　(C) 0.46 J (焦耳)　　(D) 0.52 J (焦耳)　　(E) 0.92 J (焦耳)

5. 如圖所示，一顆小衛星在橢圓軌道上繞著地球運行。若 L 表示其角動量的大小，K 表示動能。他們在1 和2位置的關係分別是：
A small satellite orbits about the Earth in an elliptical path as shown. If L represents the magnitude of its angular momentum and K represents kinetic energy, what are the relationships between these quantities at positions 1 and 2 in the orbit, respectively?

(A) $L_2 = L_1$ and (和) $K_2 > K_1$
(B) $L_2 > L_1$ and (和) $K_2 > K_1$
(C) $L_2 > L_1$ and (和) $K_2 = K_1$
(D) $L_2 = L_1$ and (和) $K_2 = K_1$
(E) $L_2 < L_1$ and (和) $K_2 = K_1$
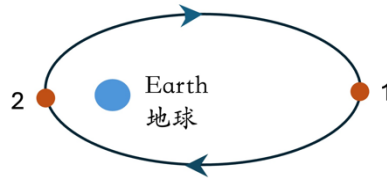
6. 球殼具有內半徑 R_1、外半徑 R_2 和質量 M，均勻分佈在整個球殼中。質量為 m 的點粒子（位於內半徑外部和外半徑內部，距離中心的距離 d）施加在球殼上的重力大小為：
A spherical shell has inner radius R_1, outer radius R_2, and mass M, distributed uniformly throughout the shell. The magnitude of the gravitational force exerted on the shell by a point particle of mass m, located at a distance d from the center, and between the inner and outer radius of the spherical shell is:

(A) 0
(B) GMm/d^2
(C) $GMm/(R_2^2 - d^2)$
(D) $GMm(d^3 - R_1^3)/d^2(R_2^3 - R_1^3)$
(E) $GMm/(d^2 - R_1^2)$

7. 如圖所示，一塊 80 N 的均勻木板靠在無摩擦的牆壁上。牆壁相對於P點施加在木板上的力矩為：
An 80-N uniform plank is leant against a frictionless wall as shown. The torque (about point P) applied to the plank by the wall is:

(A) 40 N·m (牛頓·米)
(B) 60 N·m (牛頓·米)
(C) 120 N·m (牛頓·米)
(D) 160 N·m (牛頓·米)
(E) 240 N·m (牛頓·米)
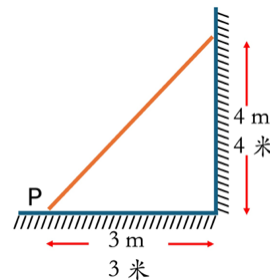
8. 兩個相同的無阻尼振盪器要具有相同的振盪振幅，只有當：
Two identical undamped oscillators have the same amplitude of oscillation only if:

(A) 他們起始於相同的位移 x_0
    they are started with the same displacement x_0
(B) 他們起始於相同的速度 v_0
    they are started with the same velocity v_0
(C) 他們起始相同的相位
    they are started with the same phase
(D) 他們起始於相同的 $\omega^2 x_0^2 + v_0^2$
    they are started with the same combination of $\omega^2 x_0^2 + v_0^2$
(E) 他們起始於相同的 $x_0^2 + \omega^2 v_0^2$
    they are started with the same combination $x_0^2 + \omega^2 v_0^2$

9. 草皮灑水器是由一端封閉直徑為 1.0 公分的軟管以及在封閉端附近鑿有 25 個小孔所組成，每個孔的直徑為 0.050 公分。如果水在軟管中的流速為 2.0 米/秒，水離開一個孔的速度為
   A lawn sprinkler is made of 1.0 cm diameter garden hose with one end closed and 25 holes, each with a diameter of 0.050 cm near the closed end. If water flows at 2.0 m/s in the hose, the speed of the water leaving a hole is:

   (A) 20 m/s (米/秒)
   (B) 32 m/s (米/秒)
   (C) 40 m/s (米/秒)
   (D) 600 m/s (米/秒)
   (E) 800 m/s (米/秒)

10. 軟木塞漂浮在靜置於電梯內裝有水的水桶中。當電梯加速向上，在加速過程中：
    A cork is floating on the water contained in a bucket which is put at rest on the floor in an elevator. The elevator is then accelerating upward. During the acceleration:

    (A) 軟木塞沉入水中的部分更多
       the cork is immersed more
    (B) 軟木塞沉入水中的部分較少
       the cork is immersed less
    (C) 軟木塞沉入水中的量不變
       the cork is immersed with the same amount
    (D) 一開始軟木塞沉的較少，加速向上時沉的比較多
       at first, the cork is immersed less but as the elevator speeds up it is immersed more
    (E) 一開始軟木塞沉的較多，加速向上時沉的比較少
       at first, the cork is immersed more but as the elevator speeds up it is immersed less

11. 熱容量 C_A 初始溫度為 T_A 的物體 A 與熱容量 C_B 初始溫度為 T_B 的物體 B 在絕熱的情形下做熱接觸。如果熱容與溫度無關且不發生相變，則兩個物體的最終溫度為：
    Object A, with heat capacity C_A and initially at temperature T_A, is placed in thermal contact with object B, with heat capacity C_B and initially at temperature T_B. The combination is thermally isolated. If the heat capacities are independent of the temperature and no phase changes occur, the final temperature of both objects is:

    (A) $(C_A T_A - C_B T_B) / (C_A + C_B)$
    (B) $(C_A T_A + C_B T_B) / (C_A + C_B)$
    (C) $(C_A T_A - C_B T_B) / (C_A - C_B)$
    (D) $(C_A - C_B) |T_A - T_B|$
    (E) $(C_A + C_B) |T_A - T_B|$

12. 通過導熱板的熱流速率為 P_c。如果導熱板厚度加倍，其橫截面積減半，兩端溫差加倍，則熱流速率變為：
    The rate of heat flow through a slab is P_c. If the slab thickness is doubled, its cross-sectional area is halved, and the temperature difference across it is doubled, then the rate of heat flow becomes:

    (A) $2 P_c$
    (B) $P_c / 2$
    (C) P_c
    (D) $P_c / 8$
    (E) $8 P_c$

13. 一定量的理想氣體被壓縮到其初始體積的一半。這個過程可以是絕熱的、等溫的或在等壓下進行。從最小到最大依序排列外力所需做的功：
    A quantity of an ideal gas is compressed to half of its initial volume. The process may be adiabatic, isothermal or occur at constant pressure. Rank these three processes in order of the work required by an external agent, from least to greatest.

    (A) 絕熱、等溫、等壓
       adiabatic, isothermal, constant pressure
    (B) 絕熱、等壓、等溫
       adiabatic, constant pressure, isothermal
    (C) 等溫、絕熱、等壓
       isothermal, adiabatic, constant pressure
    (D) 等壓、絕熱、等溫
       constant pressure, adiabatic, isothermal
    (E) 等壓、等溫、絕熱
       constant pressure, isothermal, adiabatic

14. 聲源發出頻率為 1000 Hz 的聲音。它以 20 米/秒 的速度向靜止的反射牆移動。若聲速為 340 米/秒，位於聲源正後方的觀察者聽到的拍頻為：
    A source emits sound with a frequency of 1000 Hz. It is moving at 20 m/s toward a stationary reflecting wall. If the speed of sound is 340 m/s, an observer at rest directly behind the source hears a beat frequency of:

    (A) 3.0 Hz (赫茲) (B) 55 Hz (赫茲)   (C) 63 Hz (赫茲)   (D) 118 Hz (赫茲)   (E) 183 Hz (赫茲)

15. 一端有開口與一端閉塞的兩根管子。管 A 的長度為 L，管 B 的長度為 2L。管B的哪一個諧波在頻率上與 A 管的基頻相符？
    Two pipes are each open at one end and closed at the other. Pipe A has a length *L* and pipe B has a length 2*L*. Which harmonic of pipe B matches in frequency the fundamental of pipe A?

    (A) 基頻 The fundamental
    (B) 第二諧音 The second
    (C) 第三諧音 The third
    (D) 第四諧音 The fourth
    (E) 沒有對應的諧音 There are none

16. 垂直入射的單色光照射到空氣中的薄膜。如果 λ 表示薄膜中的波長，當反射光最大時最薄的薄膜是多少？
    Monochromatic light, at normal incidence, strikes a thin film in air. If λ denotes the wavelength in the film, what is the thinnest film in which the reflected light will be a maximum?

    (A) 遠比 λ 小; much less than λ (B) $\lambda/4$ (C) $\lambda/2$ (D) $3\lambda/4$ (E) λ

17. 在楊氏雙縫實驗中，狹縫之間的間距為 d，光屏與狹縫的距離為 D。D 遠大於 d，λ 是光的波長。螢幕上單位長度的亮條紋數為：
    In a Young's double-slit experiment, the separation between slits is d and the screen is a distance *D* from the slits. *D* is much greater than d and λ is the wavelength of the light. The number of bright fringes per unit length on the screen is:

    (A) Dd/λ (B) Dλ/d   (C) D/(dλ)   (D) λ/(Dd)   (E) d/(Dλ)

18. 兩個相距很遠的導體球。較小球體的總電荷量為 Q，而較大的球體為電中性其半徑是小球體的兩倍大。以導線連接兩個球體後，較小和較大球體上的電荷分別為：

    Two conducting spheres are far apart. The smaller sphere carries a total charge of Q. The larger sphere has a radius that is twice that of the smaller and is neutral. After the two spheres are connected by a conducting wire, the charges on the smaller and larger spheres, respectively, are:

    (A) $Q/2$ and (和) $Q/2$
    (B) $Q/3$ and (和) $2Q/3$
    (C) $2Q/3$ and (和) $Q/3$
    (D) 0 and (和) Q
    (E) $2Q$ and (和) $-Q$

19. 兩個電荷 q_1 和 q_2 的點粒子放在相距 r 的位置。在連接粒子的線段上，介於粒子之間的P點電場為零。可知：

    Two point particles, with charges of q_1 and q_2, are placed at a distance r apart. The electric field is zero at a point P between the particles on the line segment connecting them. We conclude that:

    (A) q_1 and q_2 電荷同號且電量相等
     q_1 and q_2 must have the same magnitude and sign
    (B) P 點位於兩電荷中間
     P must be midway between the particles
    (C) q_1 and q_2 電荷同號但電量不一定相等
     q_1 and q_2 must have the same sign but may have different magnitudes
    (D) q_1 and q_2 電荷反號但電量相等
     q_1 and q_2 must have equal magnitudes and opposite signs
    (E) q_1 and q_2 電荷反號且電量不一定相等
     q_1 and q_2 must have opposite signs and may have different magnitudes

20. 電池提供由兩個相同電容器經由串聯組合的系統充電。若電池兩端的電位差為 V，充電過程中流經電池的總電荷 Q，則每個電容器正極板上的電荷和每個電容器兩端的電位差分別為：

    A battery is used to charge a series combination of two identical capacitors. If the potential difference across the battery terminals is V and total charge Q flows through the battery during the charging process, then the charge on the positive plate of each capacitor and the potential difference across each capacitor respectively are:

    (A) $Q/2$ and (和) $V/2$
    (B) Q and (和) V
    (C) $Q/2$ and (和) V
    (D) Q and (和) $V/2$
   (E) Q and (和) $2V$

21. 某安培計內阻為 1 歐姆，可量測範圍為 0~50 毫安培。若要使電流量測範圍變為 0 至 5 安培，需使用：

    A certain ammeter has an internal resistance of 1 Ω and a range from 0 to 50 mA (m=10⁻³). To make its range from 0 to 5 A, use:

    (A) 串聯一個 99 歐姆的電阻
       a series resistance of 99 Ω
    (B) 串聯一個很大的電阻，例如10^6 歐姆
       an extremely large (say 10^6 Ω) series resistance
    (C) 並聯一個 99 歐姆的電阻
       a resistance of 99 Ω in parallel
    (D) 並聯一個 1/99 歐姆的電阻
       a resistance of 1/99 Ω in parallel
    (E) 並聯一個 1/1000 歐姆的電阻
       a resistance of 1/1000 Ω in parallel

22. 電子（質量 m，電荷 –e）透過電位差 V 從靜止加速，之後被垂直於其速度的磁場 B 偏轉。此時電子軌道半徑為：

    Electrons (mass *m*, charge *–e*) are accelerated from rest through a potential difference *V* and are then deflected by a magnetic field $\vec{B}$ that is perpendicular to their velocity. The radius of the resulting electron trajectory is:

    (A) $\left(\sqrt{2eV/m}\right)/B$　(B) $B\sqrt{2eV}/m$　(C) $\left(\sqrt{2mV/e}\right)/B$　(D) $B\sqrt{2mV}/e$　(E) none of these (以上皆非)

23. 如圖所示，在均勻磁場 B 中將一根導體桿橫放在無摩擦軌道上。導體桿以速度 $\vec{v}$ 向右移動。為了使電路周圍的感應電動勢為零，磁場的大小應

    A rod lies across frictionless rails in a uniform magnetic field B, as shown. The rod moves to the right with speed $\vec{v}$. In order for the induced emf around the circuit to be zero, the magnitude of the magnetic field should:

    (A) 不變
       not change
    (B) 隨時間呈線性增加
       increase linearly with time
    (C) 隨時間呈線性減少
       decrease linearly with time
    (D) 隨時間呈二次方增加
       increase quadratically with time
    (E) 隨時間呈二次方減少
       decrease quadratically with time
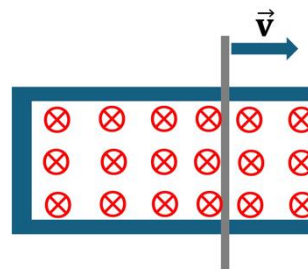
24. 下列哪一項是物質波動性的最佳證據？

    Which of the following is the best evidence for the wave nature of matter?

    (A) 電子在晶格的反射

    The reflection of electrons by crystals

    (B) 光電效應

    The photoelectric effect

    (C) 康普頓效應

    The Compton effect

    (D) 空腔輻射的光譜輻射率

    The spectral radiancy of cavity radiation

    (E) 電子的動量與能量關係

    The relationship between momentum and energy for an electron

25. 當我們觀察時，一艘太空船在時間 Δt 中從我們身邊經過。太空船的船員測量了通過時間，發現它是 $\Delta t'$。下列哪一項為真？

    As we watch, a spaceship passes us in time Δt. The crew of the spaceship measures the passage time and finds it to be $\Delta t'$. Which of the following statements is true?

    (A) Δt 是通過的固有時，比 $\Delta t'$ 小

    Δt is the proper time for the passage and it is smaller than $\Delta t'$

    (B) Δt 是通過的固有時，比 $\Delta t'$ 大

    Δt is the proper time for the passage and it is greater than $\Delta t'$

    (C) $\Delta t'$ 是通過的固有時，比 Δt 小

    $\Delta t'$ is the proper time for the passage and it is smaller than Δt

    (D) $\Delta t'$ 是通過的固有時，比 Δt 大

    $\Delta t'$ is the proper time for the passage and it is greater than Δt

    (E) None of the above statements are true (以上皆非)